

An Anxiety-Based Formulation to Estimate the Generalized Cost of Transit Travel Time

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ABSTRACT

This paper examines the impact of unreliable transit service on transit user costs with the goal of increasing the accuracy of mode choice models. The concept advanced here is to include explicitly in the formulation of mode choice models the anxiety experienced by passengers when service is unreliable due to late departure or longer than expected in-vehicle travel time. We model this anxiety as a generalized cost penalty which is added to actual in-vehicle time. The magnitude of the penalty depends on the traveler's assessment of the likelihood of arriving on time at his destination. We feel this formulation of anxiety is behaviorally representative.

To test the impacts of our formulation, we generate a simulation model that quantifies the anxiety component of generalized cost for 10,000 travelers with various aversions to risk for travel between station pairs with different observed reliabilities. Our results suggest that primarily for risk averse travelers, but also for other classes, anxiety may constitute very high percentages of total generalized cost which may explain many travelers' unwillingness to choose transit in cases where deterministic models suggest they will.

Calibrating a model of this type presents substantial challenges. We introduce an approach that we are currently pursuing to gather actual anxiety levels as a function of transit travel reliability. We conclude with comments on future research directions.

INTRODUCTION

In the past decade, the importance of public transportation in achieving balanced transportation has increased throughout North America. In Ontario, Canada, for example, the Provincial Government is proposing transit projects in excess of \$17.5B prior to 2020 (1). As a result, models that are used to estimate transit ridership have gained increasing attention from transportation planners and engineers. Though various mode choice models exist, nearly all are based on a comparison of user cost (or disutility) for travel by available modes. In order for the models to be effective, the modal costs should be representative of the “real-world” costs experienced by users.

The way in which transit user costs are typically represented is through generalized costs which are calculated as a linear sum of monetary costs (the fare) and a series of travel time components converted to monetary units assuming a value of time. Most commonly included times in a generalized cost equation are access to the transit system, waiting for a transit vehicle, in-vehicle, transfers (where appropriate) and sometimes egress from the vehicle to the final destination.

In conventional generalized cost formulations, transit costs are most frequently considered to be deterministic. While a deterministic representation of access time is probably sufficient, it is quite evident that great variability exists in waiting times and in-vehicle times and, as such, total costs experienced by transit users. Generalized cost models should be constructed to incorporate stochastic cost components for waiting and in-vehicle times.

The limiting factor to developing stochastic models of generalized costs was the availability of data for on-time system performance. Given the increase in the use of Automatic Vehicle Location (AVL) for bus systems, it has become much less difficult to produce distributions of these travel time components. The stochasticity of travel cost components have been reflected by Bates (2) Casello *et al.* (3) in mode choice models. These formulations include penalties for arriving prior to or later than a desired arrival time with the likelihood of experiencing arrival penalties estimated from empirically derived observations of transit service reliability.

A second source of stochasticity in generalized cost models is derived from users’ cost perceptions which may differ based on trip purpose or, alternatively traveler characteristics (risk averse, risk neutral or risk tolerant). This variability in users’ perceptions and therefore generalized cost has been modeled by defining penalties as a function of traveler class.

In this paper, we extend previous models to investigate further users’ perceptions of in-vehicle time. The model is grounded in the idea that unreliable departure times or in-vehicle times create passenger anxiety which influence traveler perception of in-vehicle time. The concept developed here formally presents a methodology to complement typical generalized cost components with “anxiety costs” which are computed based upon stochastic departure times, in-vehicle times, and potential late penalties for various traveler classes. We suggest that this measurement of anxiety in light of stochastic travel components better represents traveler perception and is therefore more behaviorally representative than previous efforts.

The remainder of the paper is organized as follows. We first briefly review the literature to document previous efforts at incorporating reliability in transit generalized cost formulations. We next present the theoretical motivation for and the details of the methodology developed. We then demonstrate a simplified example using data from the Regional Municipality of Waterloo. After the case study, we describe the efforts necessary to calibrate a model of this type. We conclude with comments on limitations and future research needs.

LITERATURE REVIEW

Mode choice models based on user cost have been used for decades. A history and detailed explanation of these models' formulation, calibration and solution are given by Ortuzar and Willumsen (4). The basic formulation of generalized cost for transit used in this paper models that by Kittelson *et al.* (5). More detailed generalized cost models, upon which we rely heavily, explicitly treat the penalties associated with discrete departure times of transit vehicles (2) as well as unreliable service(6).

This paper builds heavily upon work done by the authors in 2009 (3). For a much more detailed literature review, the reader is directed to the lead author's most recent publication (7).

The concept of anxiety as a travel cost has been discussed by Van der Waard (8), Hickman and Wilson (9) and Bhat and Sardesai (10) amongst others. Anxiety has been explicitly considered as part of a boundedly rational model proposed by Mahmassani and Liu (11) in describing auto route choice. Guiver (12) describes qualitatively the concept of travel anxiety on public transport. We are unaware of any study that proposed to quantify the direct cost of anxiety in unreliable transit networks.

METHODOLOGY

As noted above, nearly all transit generalized cost functions assume in-vehicle time is least onerous, and constant. In our model, we relax this assumption as follows. We assume that the traveler disaggregates in-vehicle time as scheduled in-vehicle time (*SIVT*) and unscheduled in-vehicle time (*LATE*). Mathematically, *LATE* is computed as:

$$LATE = AIVT - SIVT \quad \text{Eq. 1}$$

Where *AIVT* is the actual in-vehicle time and all units are minutes. If *AIVT* is less than *SIVT*, then *LATE* is less than 0, or an early arrival. To these two components, we add a third term, *ANX*, that reflects anxiety felt in cases where service is behind schedule.

If we assume the traveler chooses a bus departure for which the scheduled arrival time (*SAT*) at his destination is earlier than his necessary arrival time (*NAT*), we can evaluate the following scenarios:

1. If that traveler boards the vehicle at or before the scheduled departure time (*SDT*), on very reliable systems, the traveler can be reasonably certain that he will arrive on time. In unreliable systems, despite boarding exactly on time, the traveler still has some uncertainty that the in-vehicle travel time will be sufficiently short to meet his necessary arrival time.
2. If the actual in-vehicle time (*AIVT*) is less than or equal to the scheduled in-vehicle time (*SIVT*), then throughout the trip the passenger has decreasing concern about arriving late.

On the reliable system, the passenger may feel nearly zero anxiety. On the unreliable system, the traveler feels slightly higher anxiety on early segments of the trip.

3. If the bus departs later than the *SDT*, then the traveler begins the in-vehicle portion of the trip with some anxiety about his likelihood of arriving on time. Subsequent trip segments are perceived as “higher cost” than the case where the bus departs on time.
4. Recurringly during the trip (say at each stop), the traveler evaluates his likelihood of arriving on time based on scheduled travel time and historical knowledge. If the bus “catches up” (recovers lost time) and regains the scheduled time, then the traveler’s anxiety lessens and is eventually eliminated. In this case, the traveler experiences a shorter than expected *SIVT* and no late penalty. In our formulation, however, we explicitly quantify the cost associated with the anxiety felt on the early segments of the trip.
5. If the bus remains or falls further behind schedule, the traveler’s anxiety grows during the trip until at some point, there is 0 probability of arriving on time. At this point, the anxiety for subsequent sections is constant at the maximum value.

The travel time and perceived anxieties for scenarios 2, 4 and 5 are shown in Figure 1. Here, the bus route contains an origin, *O*, a destination, *D*, and two intermediate stations *Z*, and *Y*. The top part of the figure shows the time distance diagram of the bus relative to the *SDT*, the *SAT* and the *NAT*. The bottom part of the figure shows the probability of late arrival under two conditions: generally reliable bus service (*R*) and unreliable service (*UR*). Note that the probability of being late, and therefore passenger anxiety, is always lower in the reliable service case.

In the first case, the bus departs precisely at the scheduled departure time and the in-vehicle travel time is equal to the scheduled time. The arrival time at the destination occurs earlier the necessary arrival time. Anxiety begins with a positive value, but decreases after each station because the traveler recognizes the service as on time.

In the second case, the bus departs late, resulting in higher initial anxiety than in the previous case. During the trip, however, the bus is able to recover time. At each station, the traveler’s anxiety is reduced again because he recognizes that his probability of late arrival is decreasing. As in the first case, the actual arrival time coincides with the schedule arrival time and the final probability of late arrival is zero.

In the final case, the bus both departs late and experiences longer than expected travel times. In this case, the initial anxiety is as high as in the previous case. As the trip progresses, the likelihood of an on-time arrival decreases and, the probability of late arrival increases towards one. By the time the bus reaches station *Y*, he is certain that an on-time arrival is not possible.

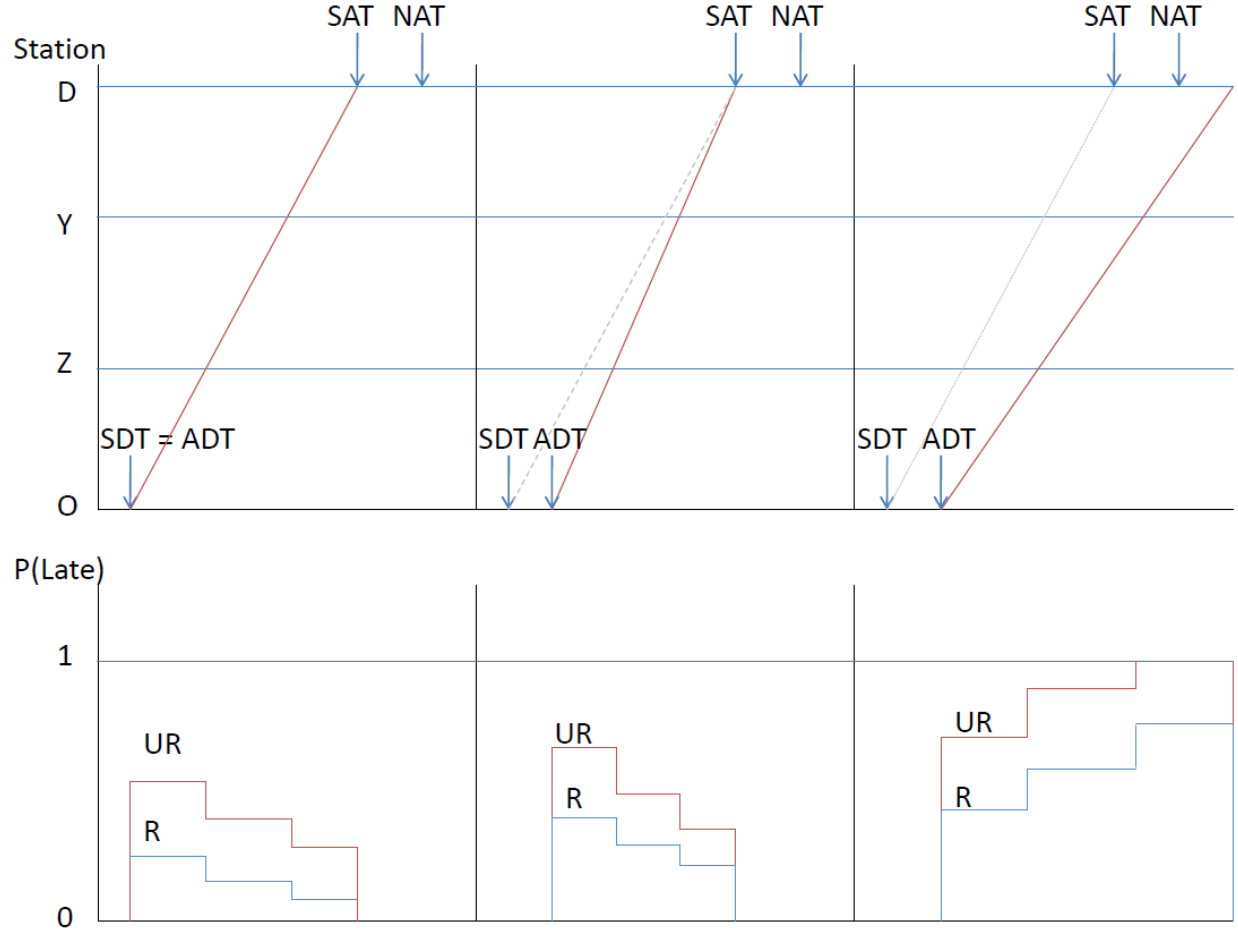


FIGURE 1 The Relationship between Departure Time, In-vehicle Time and Anxiety

To quantify the costs of traveler anxiety we propose the following method. We assume the traveler assesses his likelihood of on-time arrival at each stop from boarding to destination along the route which, in turn, generates a level of anxiety. We further assume that the traveler's perception of the next inter-station in-vehicle travel time is dependent on the anxiety felt at the previous stop. We calculate the anxiety cost on a segment from station i to station $i+1$ as the product of the probability of being late calculated at station i and the actual travel time from i to $i+1$.

Mathematically, this is expressed as:

$$ANX_{i,i+1} = P_{late}^i \cdot AIVT_{i,i+1} \quad \text{Eq. 2}$$

To compute the total anxiety experienced over the entire trip (from origin O, to destination D), we sum over all stations from origin to destination.

$$ANX_{OD} = \sum_{i=0}^{D-1} P_{late}^i \cdot AIVT_{i,i+1} \quad \text{Eq. 3}$$

Equation 3 reflects the computation of the areas under the curves shown in the lower half of Figure 1.

The total contribution of in-vehicle cost to generalized cost with unreliable service, GC_R , is then the sum of $SIVT$, $LATE$ and the anxiety penalty:

$$GC_R(AIVT_{OD}) = SIVT_{OD} + \alpha LATE_{OD} + \beta ANX_{OD} \quad \text{Eq. 4}$$

Where α and β are calibration parameters that weigh the relative importance of each component. As discussed above, in the case where $AIVT < SIVT$, then α defaults to 1 and $LATE$ is actually negative, reducing the generalized cost of in-vehicle travel time.

Our anxiety measure depends upon a traveler's ability to assess the likelihood of being late from his current point to the destination, given the travel time experienced from the origin to his current point. In reality, this depends upon a traveler's familiarity with the system and historical knowledge of expected travel times. In our model, we compute directly the probability of arriving late to the destination having traveled to each station i . This probability is the likelihood of experiencing a travel time from i to D that exceeds the difference between the NAT and the arrival time at i ; the arrival time at i is given by the sum of the actual departure time from the origin (ADT_O) and the actual in-vehicle time to i , ($AIVT_{O,i}$). Mathematically, this is expression is:

$$P_{late}^i = P(AIVT_{i,D} \geq NAT - (ADT_O + AIVT_{O,i})) \quad \text{Eq. 5}$$

All terms on the right hand side of the inequality - the necessary arrival time, the actual departure time from the origin, and the travel time from the origin to i - are known. As such, the probability of an on-time arrival is simply given by the probability of a travel time from i to D less than a constant. This is a cumulative probability distribution function for travel time between stations. Using Automatic Vehicle Location (AVL) data, it is a straightforward exercise to derive these CDFs from which we can easily solve equation 5 for any station i along the route.

Figure 2 displays the logic of the approach graphically. At the origin, there is a distribution of potential departure times for the service around scheduled departure time. There is also a distribution of actual travel times from O to D , shown on the right hand side of Figure 2. The likelihood of being late with an on-time departure can be seen by area under the dashed distribution that exceeds the NAT . In this case, the actual departure time of the vehicle from the origin occurs sometime later than the scheduled time. This shifts the distribution of travel times to the right by the same amount as the lateness of departure which increases the likelihood of being late. This is shown by the increase in area under the solid curve which falls to the right of the necessary arrival time along the O line.

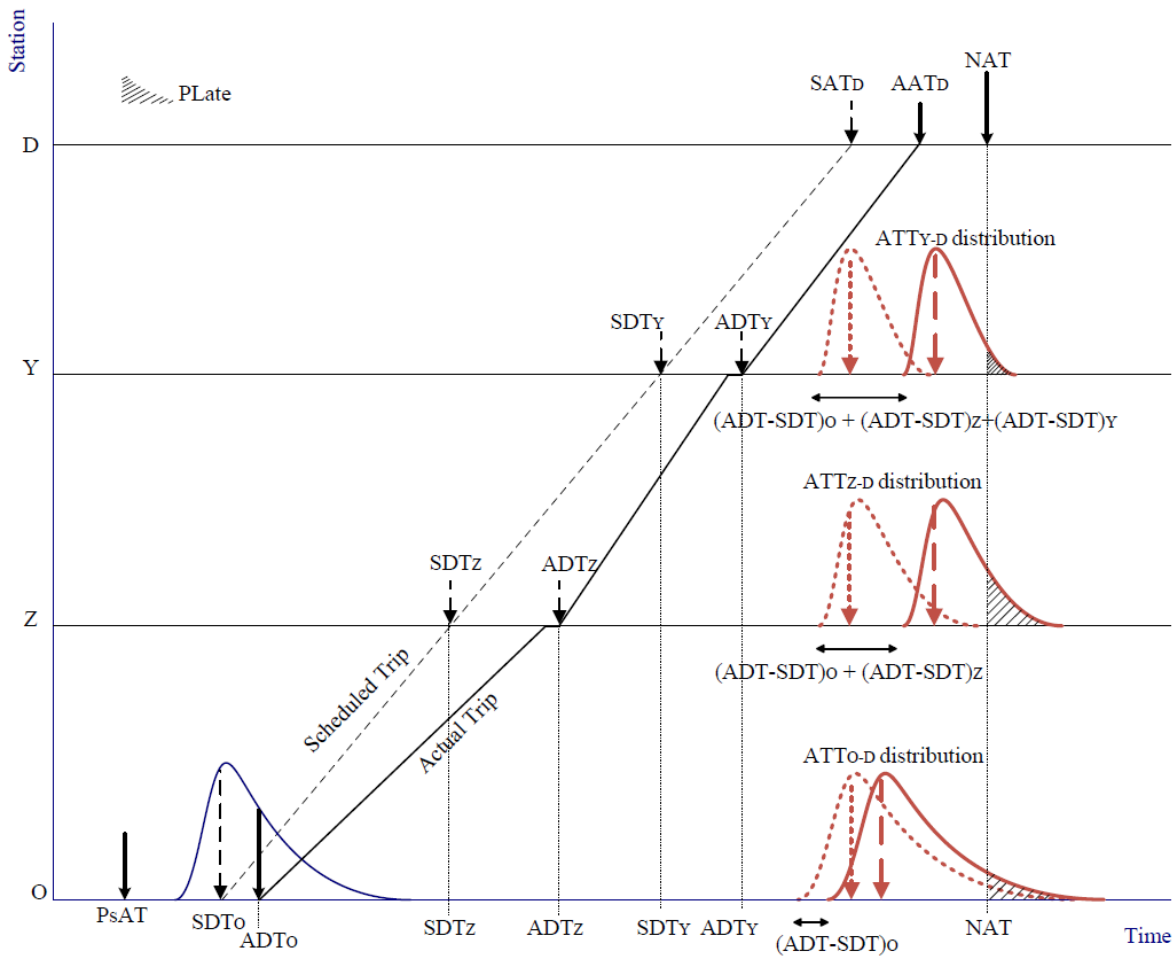


FIGURE 2 Graphical Representation of Probability Functions

Moving up from the origin to station Z, two characteristics change. First, the travel time distribution from Z to D narrows relative to the travel time distribution from O to D reflecting the shorter distance remaining to travel. Second, because the service has fallen further behind from O to Z, the distribution of travel times from Z to D has shifted right by the amount the service is late, increasing the probability of a late arrival. At Y, the distribution once again narrows due the proximity to the destination; also from Z to Y, the bus recovers time, shifting the travel time distribution left and decreasing the likelihood of late arrival.

APPLICATION OF METHODOLOGY

To test the implications of the above formulation, we develop a simulation model for the bus system serving the Regional Municipality of Waterloo (Ontario). The Region's transit operator, Grand River Transit (GRT), runs iXpress service, a 33 km limited-stop route with 13 stops. The alignment, shown in Figure 3 connects four downtowns - Waterloo, Kitchener and two in Cambridge - as well as two universities, office complexes, major hospitals and regional shopping centers. iXpress operates throughout the day with 15 minute headways and vehicles are equipped with AVL technology.

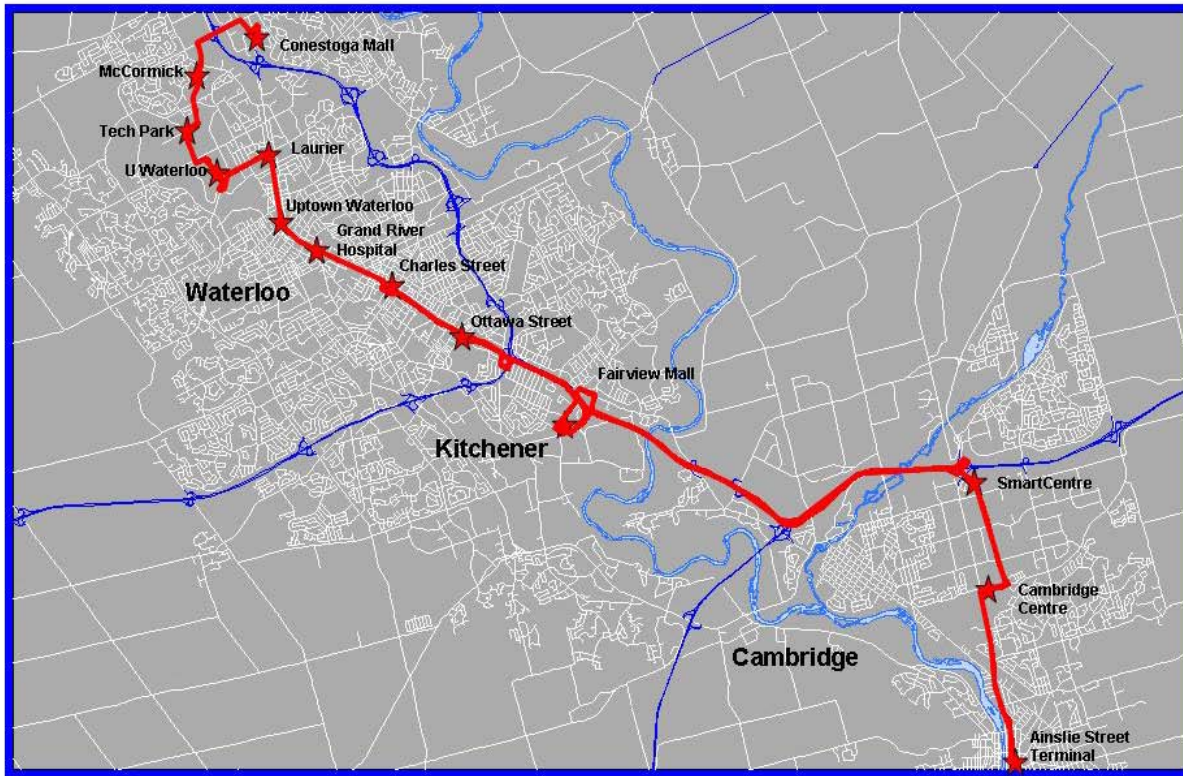


FIGURE 3 iXpress Route Serving Waterloo, Kitchener and Cambridge

Model Formulation

We are interested in understanding how reliability (or the lack thereof) influences users' perception of in-vehicle travel time, and how those perceptions should be reflected in generalized costs. To this end, we simulate 10,000 peak hour trips between two origin destination pairs. The first trip begins at the Charles St. Terminal in Kitchener and ends at the route's southern terminal, Ainslie St. Including the boarding station, there are five locations at which the probability of being late must be evaluated. The second trip considered originates at Fairview Mall traveling northbound to Conestoga Mall, with nine intermediate stops. Table 1 shows a statistical representation of travel times between each station pair derived from more than 600 actual AVL observations of travel time.

Table 1 Statistical Parameters for the Station Pairs Modeled

Statistical Characteristic	Charles St. to Ainslie	Fairview to Conestoga
Scheduled travel time (min)	42	43
50 th percentile difference between <i>SDT</i> and <i>ADT</i> (min)	0.40	0.40
90 th percentile difference between <i>SDT</i> and <i>ADT</i> (min)	1.92	4.62

The data in Table 1 suggest that while 50% of buses depart both Charles St. and Fairview with 0.40 minutes (24 seconds) of the scheduled departure time. However, the departures from

Fairview have much greater variance than those from Charles St. Only 10% of the buses leaving Charles St. do so more than 1.92 minutes after the scheduled departure time. In contrast, 10% of buses leaving Fairview towards Conestoga leave more than 4.62 minutes late. This may indicate that passengers leaving Fairview experience greater anxiety on early segments of their trips.

For each of the 10,000 trips, we generate travelers with one of three different but equally likely trip characteristics. A traveler may be risk averse, moderately risk averse or risk neutral. The perception of potential delays decreases in severity with decreasing risk aversion. As such, the values for α (when $LATE > 0$) are 3, 2 and 1 for risk averse, moderately risk averse and risk neutral passengers respectively. Similarly, the values for β are 2.5, 1.85 and 1.0 in the same order of passenger types.

For each traveler, we next estimate necessary arrival times that are uniformly distributed between two subsequent arrivals at the destination station. To model bus service, we use observed AVL data to generate the following parameters in the simulation model:

1. Actual departure time from the origin station;
2. Actual travel time between each set of stops;
3. Probability of being late at the destination given the actual departure time, the travel time from the origin to the current stop, and the necessary arrival time.

Model Results

For each traveler, the model outputs the *AIVT*, the corresponding late penalty (or benefit), the probabilities of late arrival computed at each stop, and the total generalized cost considering reliability, GC_R . We present three comparisons. First, we demonstrate how the range of observed GC_R values compare to the conventional GC_C estimates that are deterministic. The results for two traveler cases, the Risk Averse Person (RAP) and the Risk Neutral Person (RNP) are shown on the left and right respectively for the Fairview trip in Figure 4 below. The heavy vertical lines represent GC_C . The intervals represent eight units of generalized cost.

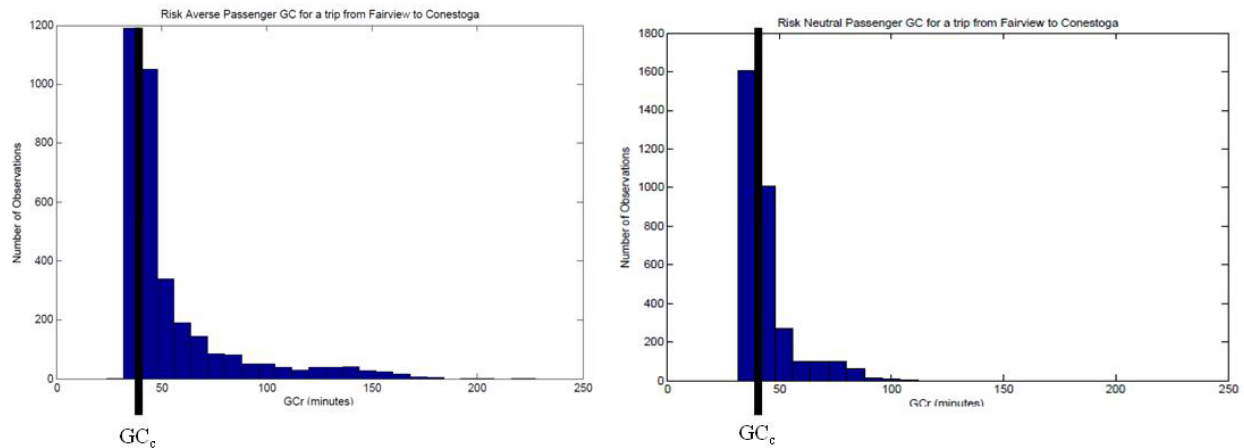


Figure 4 Comparing Generalized Cost with and without Reliability Considerations

The model predicts approximately 3300 RAP and RNP travelers. From the figure, we see that approximately 2300 RAP (or 70%) experience GC_R that are within one interval of the GC_C . Similarly, 2600 RNP travelers are within the same tolerance. However, we notice that for RAP,

there exists a long tail to the right, indicating that for many RAP travelers, the potential exists to experience very unreliable, very high cost travel by transit. In fact, our model predicts that 758 RAP travelers experiences GC_R in excess of 60, or 50% higher than what would be predicted by GC_C . For RNP, the number of passengers experiencing very high GC_R is less, about 312, but still not insignificant.

Next, we demonstrate how GC_R differs for the same traveler type, Risk moderate passengers (RMP), for the two station pairs selected for analysis. Figure 5 shows the range of GC_R results for the lower reliability pair (Fairview to Conestoga) on the left and the higher reliability pair (Charles St. to Ainslie) on the right. Again, the intervals displayed represent eight minutes.

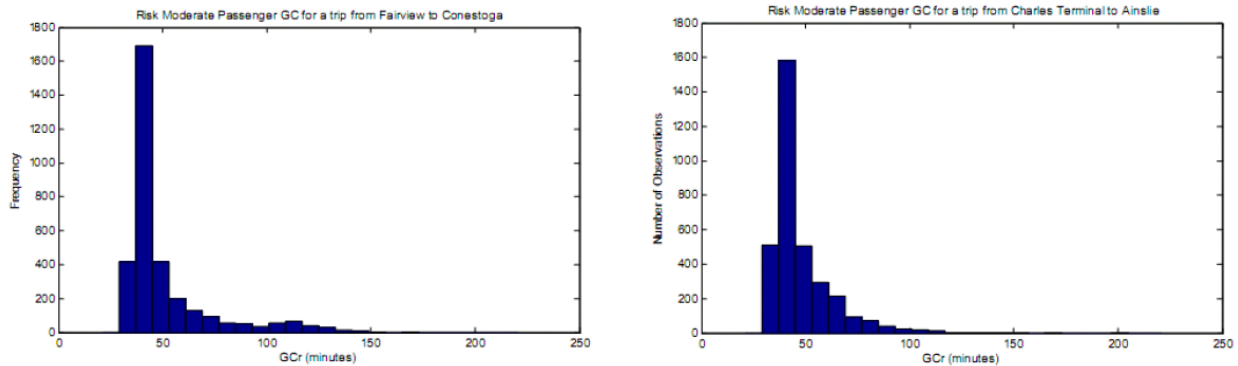


Figure 5 Reliability Influences on Travelers with Different Risk Tolerances

For lower values of GC_R , the frequencies of occurrence are very similar. Both station pairs show high concentrations of observations (approximately 2500 in both cases) within one interval of the expected GC_R . It is evident, however, that the unreliable travel pair again has many more observations of very high GC_R when compared to the higher reliability case. In fact, the number of GC_R observations exceeding 80 (or approximately double the deterministic GC_C) is 330 (10% of trips) in the unreliable case, but only 152 (approximate 5%) for the reliable travel pair.

Finally, it is interesting to understand the relative importance of the anxiety component to overall GC_R . To this end, we plot in Figure 6 a histogram of the percent of total GC_R attributable to anxiety for RAP (left) and RNP (right) travelers. In this case, the intervals are 5%.

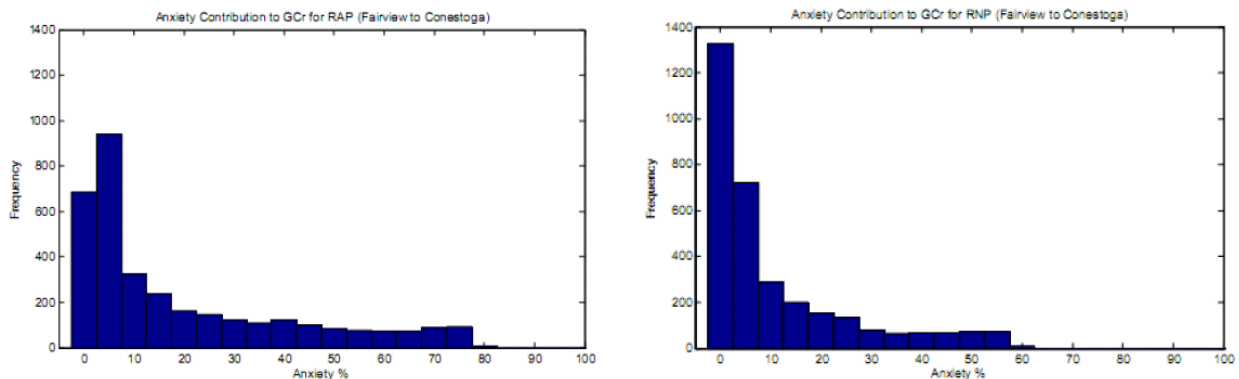


Figure 6 Percent of Total Generalized Cost Attributable to Anxiety

As in the previous example, anxiety has much greater impacts on risk averse passengers than on risk neutral. For nearly 2,000 RAP, anxiety constitutes 10% or less of the total GC_R while for RNP, that number increases to more than 2400. Also similar to the previous example, the RAP have a much longer right tail of the distribution, indicating that in many cases, 458 or nearly 14%, anxiety is the primary component of GC_R . For only 125 RNP, anxiety penalties exceed 50% of GCR ; anxiety never constitutes more than 70% of GC_R for RNP.

CALIBRATING THE MODEL

The model results presented above make assumptions about the relative importance of longer than expected in-vehicle time and the anxiety felt as the likelihood of late arrivals increases. At this moment, we have little basis for the relative weights assumed other than they are within the range of values presented in the literature (5) for other travel time components and they follow the logic that a risk averse traveler would perceive late arrivals as less tolerable than a risk moderate or risk neutral passenger. Calibration of the magnitude of the anxiety penalty is obviously necessary to make functional the model form we present here.

In order to gather data on in-vehicle anxiety levels, we propose to utilize GPS enabled, hand held devices to query bus passengers regarding current levels of dissatisfaction. These devices can be programmed with all transit schedules; their GPS capabilities allow the devices to identify the current route and schedule adherence without user input (13). We are in the process of developing code through which the hand held device recognizes stop locations at which it computes the existing schedule adherence and probability of on-time arrival. The user will automatically be prompted by the device to indicate his current level of anxiety with the route performance. The data gathering may include simple drop-down menus with quantitative inputs. One example question may be to ask respondents to indicate their expected “likelihood of on-time arrival” and provide options ranging from 0%, 20%, 40% through 100%. These user perceptions can then be directly compared to the “actual” likelihood derived from empirical observations. The differences between perception and reality help to inform the nature of a transit rider’s anxiety.

Alternatively, users may be prompted to provide qualitative indicators of anxiety using voice recording. A range of anxiety levels may be provided from, for example, no anxiety to low anxiety or very high anxiety. When a sufficient number of observations are recorded for various trip purposes, the user perception of anxiety can be mapped to the “actual” probability of late arrival (again derived from empirical data). These relationships can be used to correlate trip purpose, trip maker characteristics, on-time performance and anxiety levels experienced which when analyzed together inform the total generalized cost formulations.

CONCLUSIONS

Conventional generalized cost models fail to account for the impacts of reliability in attracting or dissuading transit riders. We supplement previous work with a new, behaviorally based model that explicitly quantifies the impacts associated with traveler anxiety generated by unreliable transit service. We use a simulation model to compare the generalized cost estimates that consider reliability versus conventional results for various traveler types. Our results suggest that for risk intolerant travelers, unreliability in transit service can produce very high costs which are likely to dissuade these individuals from choosing transit. Output from the simulation model

also suggests that small changes in travel time reliability produce the potential for very high travel costs for subsets of the population. We demonstrate that while anxiety in most cases accounts for only 10% of the total generalized cost, in approximately 5-10% of cases, anxiety may constitute more than 50% of total costs.

We recognize the complexity and difficulties associated with developing and calibrating models of traveler behavior. We describe a survey technique that employs GPS enabled hand held devices to gather sufficient data on traveler anxiety to develop meaningful weights for the anxiety component of our generalized cost model.

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